Adaptive scheme for synchronization-based multiparameter estimation from a single chaotic time series and its applications

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Chaos-synchronization-based multiparameter estimation of a multiply delayed feedback system is investigated. We propose an adaptive method that can estimate all the parameters of the response system using the driving signal only. In the past few years, various methods have been developed for estimation of multiparameters of a chaotic system but most of them require more than one time series to estimate all the parameters of a chaotic or hyperchaotic system. The proposed method requires only a single chaotic time series to estimate all the parameters. A sufficient condition for synchronization is derived and it is shown that the numerical results well support the analytic calculations. The synchronized system has applications in cryptographic encoding for digital and analog signals, which is shown with an example.

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I. INTRODUCTION

An important issue in time series analysis of chaotic systems is the estimation of one or multiple parameters from a single or multiple time series. Parameter estimation methods may be broadly classified into online and offline estimation methods. The online estimation methods, e.g., the adaptive control approach [1], though simple to implement, cannot be used to estimate all the system parameters. The offline methods, e.g., the active-passive-decomposition method [2], autosynchronization [3], the error minimization strategy [2], statistical methods [4], and iterative methods [5], have been demonstrated in many publications to estimate all the system parameters. Recently, Konnur [6] proposed online estimation for all model parameters of a given chaotic-hyperchaotic system using the least-squares approach. This method is more analytical and effective but requires more than one scalar time series to estimate all parameters of a system. Chen and Kurths [7] proposed an observer-based approach for chaos synchronization and parameter estimation from a scalar output signal. This signal could be either a variable from the master system or a scalar nonlinear combination of all variables from the master system. Adaptive chaos-synchronization-based parameter estimation was recently developed by Huang *et al.* [8]. Yu *et al.* [9] pointed out a linear independence condition which is sufficient for parameter identification in general dynamical systems. In the past few years, various methods have been developed for estimation of multiple parameters of a chaotic system and used to obtain secure communication [8]. Some methods are analytical and effective but most of them require more than one time series to estimate all the parameters of a chaotic-hyperchaotic system [2–6,10–13]. In practice this requirement increases implementation costs. Therefore, multiparameter estimation and multiparameter modulation for secure communication using only a single scalar chaotic time series are still relevant.

In this paper, we try to show that it is possible to estimate more than one parameter of a chaotic system by an adaptive method using a single chaotic time series. We use a constant function and analog and chaotic signals as modulations of various parameters of a chaotic system and prove that it is possible to recover the information using a single scalar chaotic time series. The resulting transmitted signal consists of information hidden in the signal from the chaotic system.

The structure of this paper is the following. In Sec. II, we explain the synchronization phenomenon for a multidelayed chaotic system and establish a sufficient condition for synchronization using the Krasovskii-Lyapunov approach. A scheme for multiparameter estimation is also investigated. In Sec. III, we consider two different multidelayed Ikeda models [14–17] with modulated time delay. The numerical calculations show the effectiveness of the analytical results and examples are given for encryption and decryption of signals from a synchronized system with the help of parameter estimation. Finally, in Sec. IV we summarize our results.

II. MULTIPARAMETER ESTIMATION SCHEME

In this section, we study analytically the sufficient condition for complete synchronization between two multiple delayed feedback systems with modulated delay time. Consider the chaos-synchronization-based multiparameter estimation scheme for a multiple-delay feedback system with variable time delays. The coupled system is of the form

$$\dot{x} = -ax + \sum_{i=1}^{N} m_i f(x_{\tau_i}), \quad (1)$$

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$$\dot{y} = -ay + \sum_{i=1}^{N} n_i f(y_{\tau_i}) + k(x-y), \tag{2}$$

where m_i are estimated parameters, n_i are free or adjustable parameters, and k is the coupling strength. Our aim is to devise an algorithm to adaptively adjust n_i until $y \to x$, i.e., both $y \to x$ and $n_i \to m_i$. In this way, synchronization between systems (1) and (2) is achieved and the parameters m_i are estimated.

We take τ_i, a function of time, as

$$\tau_i(t) = \tau_0 + \alpha_i \sin(\omega_i t), \quad i = 1,2, \ldots, N, \tag{3}$$

where τ_0 is the zero-frequency component, α_i are the amplitudes, and $\omega_i/2\pi$ are the frequencies of the modulation.

We propose a parameter estimation algorithm as

$$\dot{x} = -ax + \sum_{i=1}^{N} m_i f(x_{\tau_i}), \tag{4a}$$

$$\dot{m}_i = 0, \tag{4b}$$

and

$$\dot{y} = -ay + \sum_{i=1}^{N} n_i f(y_{\tau_i}) + k(x-y), \tag{5a}$$

$$\dot{n}_i = g_i, \tag{5b}$$

where g_i are the adaptive functions to be determined. Next we want to find the functions g_i with the help of the Krasovskii-Lyapunov functional approach.

Let $\Delta = x - y$ and $e_i = m_i - n_i$ be the synchronization error and parameter error, respectively. Then the dynamics of the error are

$$\dot{\Delta} = -(a+k)\Delta + \sum_{i=1}^{N} n_i f'(y_{\tau_i})\Delta(t-\tau_i) + \sum_{i=1}^{N} f(x_{\tau_i}) e_i,$$

$$\dot{e}_i = -g_i.$$

We define a positive definite Krasovskii-Lyapunov functional [15–17] of the form

$$V(t) = \frac{1}{2}\Delta^2 + \frac{1}{2}\sum_{i=1}^{N} e_i^2 + h(t)\sum_{i=1}^{N}\int_{-\tau_i(t)}^{0} \Delta^2(t+\theta)d\theta.$$

Then

$$\dot{V}(t) = \Delta\dot{\Delta} + \sum_{i=1}^{N} e_i \dot{e}_i + \dot{h}(t)\sum_{i=1}^{N}\int_{-\tau_i(t)}^{0} \Delta^2(t+\theta)d\theta + h(t) \times \sum_{i=1}^{N} [\Delta^2 - \Delta^2(t-\tau_i) + \Delta^2(t-\tau_i)\tau_i'(t)].$$

If $\dot{h}(t) \leqslant 0$ for arbitrary t, then

$$\dot{V}(t) \leqslant -(a+k)\Delta^2 + \sum_{i=1}^{N} n_i f'(y_{\tau_i})\Delta\Delta(t-\tau_i) + h(t)\Delta^2 + \sum_{i=1}^{N} [\dot{e}_i + f(x_{\tau_i})\Delta]e_i - h(t)\sum_{i=1}^{N}(1-\tau_i')\Delta^2(t-\tau_i).$$

If we choose $\dot{e}_i = -f(x_{\tau_i})\Delta$, $i = 1, 2, \ldots, N$, then

$$\begin{aligned}\dot{V}(t) &\leqslant -[a+k-h(t)]\Delta^2 - h(t)\sum_{i=1}^{N}(1-\tau_i')\Delta^2(t-\tau_i) + \sum_{i=1}^{N} n_i f'(y_{\tau_i})\Delta\Delta(t-\tau_i) \\ &= -[a+k-h(t)]\Delta^2 + \frac{n_i^2 f^{2\prime}(y_{\tau_i})}{4h(t)(1-\tau_i')}\Delta^2 - h(t)\sum_{i=1}^{N}(1-\tau_i') \times \left(\Delta(t-\tau_i) - \frac{n_i f'(y_{\tau_i})}{2h(t)(1-\tau_i)}\Delta\right)^2 \\ &< -\left(a+k-h(t) - \frac{1}{4h(t)}\sum_{i=1}^{N}\frac{n_i^2 f'^2(y_{\tau_i})}{1-\tau_i'}\right)\Delta^2 \\ &= -F(h(t),Q)\Delta^2,\end{aligned}$$

where $Q = \Sigma_{i=1}^{N} n_i f'^2(y_{\tau_i})/(1-\tau_i')$ and $F(h(t),Q) = a + k - h(t) - Q/4h(t)$. In order to prove that $\dot{V}(t) < 0$, it is sufficient to show that $F_{\min} > 0$. This occurs for $h(t) = \sqrt{Q}/2$ with $F_{\min} = a + k - \sqrt{Q}$. Finally, we get the sufficient condition for synchronization as

$$a + k > \left(\sum_{i=1}^{N}\frac{|n_i||\sup f'^2(y_{\tau_i})|}{1-\tau_i'}\right)^{1/2}. \tag{6}$$

Note that Eq. (6) is also the condition for communicating the information messages. If Eq. (6) is satisfied then we can transmit any finite number of information signals through a single chaotic channel.

III. NUMERICAL SIMULATION

In this section we want to verify the analytic calculations done in Sec. II with some numerical methods. We consider a coupled Ikeda system [14] with one time delay as

$$\dot{x} = -ax + m_1 \sin x_{\tau_1(t)}, \tag{7}$$

and

$$\dot{y} = -ay + n_1(t)\sin y_{\tau_1(t)} + k(x-y), \tag{8}$$

$$\dot{n}_1(t) = (x-y)\sin x_{\tau_1(t)}. \tag{9}$$

We choose the parameter values as $a = 1.0$, $m_1 = 4.0$, $\tau_0 = 2.0$, $\alpha_1 = 0.05$, and $\omega_1 = 0.0001$. Here $m_1 = 4.0$ is the only estimated parameter. From condition (6), one can obtain the sufficient condition for synchronization as $k > 2.000\,005$. For our numerical simulation we choose $k = 2.01$; the variation of $n_1(t)$ and synchronization error $\Delta = x - y$ are shown in Fig. 1(a).

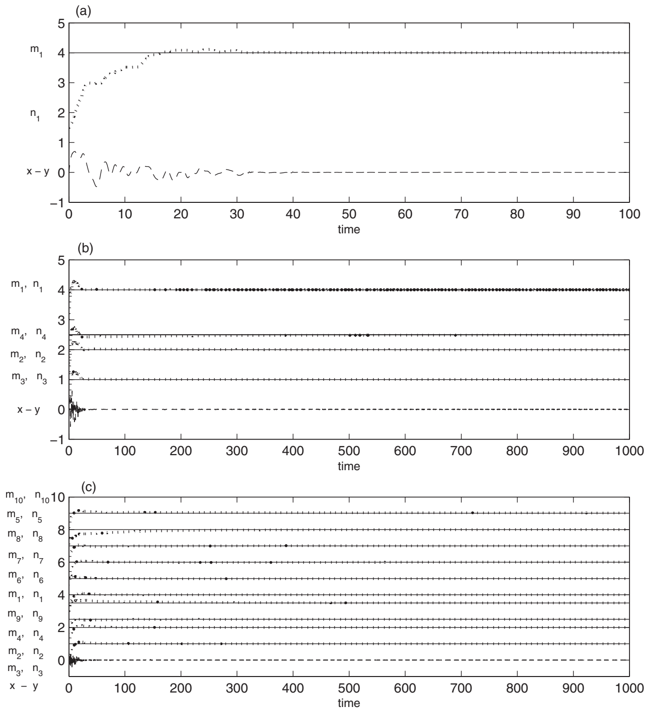

FIG. 1. Variation of original parameter (solid lines), estimated parameter (dotted lines), and synchronization error (dashed lines) for (a) one, (b) four, and (c) ten parameters.

Here m_1 is represented by the solid lines, $n_1(t)$ by the dotted lines, and the synchronization error by the dashed line. To check the above analytical condition for a multiple-delay system, we consider the coupled Ikeda system with four variable time delays as

$$\dot{x} = -ax + \sum_{i=1}^{4} m_i \sin x_{\tau_i(t)}, \tag{10}$$

and

$$\dot{y} = -ay + \sum_{i=1}^{4} n_i(t) \sin y_{\tau_i(t)} + k(x-y), \tag{11}$$

$$\dot{n}_i(t) = (x-y)\sin x_{\tau_i(t)}, \quad i = 1,2,3,4. \tag{12}$$

We choose the parameter values as $m_1=4.0$, $m_2=2.0$, $m_3=1.0$, $m_4=2.5$, $\alpha_1=0.05$, $\omega_1=0.0001$, $\alpha_2=0.01$, $\omega_2=0.001$, $\alpha_3=0.1$, $\omega_3=0.0001$, $\alpha_4=0.06$, and $\omega_4=0.1$. We take $k=5.0$ which satisfies the condition (6). The corresponding parameter variations and their errors are shown in Fig. 1(b). Solid, dotted, and dashed lines are for m_i, $n_i(t)$, and $x-y$, respectively.

For a ten-parameter estimation, we consider the coupled system as

$$\dot{x} = -ax + \sum_{i=1}^{10} m_i \sin x_{\tau_i(t)}, \tag{13}$$

and

$$\dot{y} = -ay + \sum_{i=1}^{10} n_i(t) \sin y_{\tau_i(t)} + k(x-y), \tag{14}$$

$$\dot{n}_i(t) = (x-y)\sin x_{\tau_i(t)}, \quad i = 1,2,\ldots,10. \tag{15}$$

The parameters and delays are chosen as $m_1=4.0$, $m_2=2.0$, $m_3=1.0$, $m_4=2.5$, $m_5=8.0$, $m_6=5.0$, $m_7=6.0$, $m_8=7.0$, $m_9=3.5$, $m_{10}=9.0$, $\alpha_1=0.05$, $\omega_1=0.0001$, $\alpha_2=0.01$, $\omega_2=0.001$, $\alpha_3=0.1$, $\omega_3=0.0001$, $\alpha_4=0.06$, $\omega_4=0.1$, $\alpha_5=0.11$, $\omega_5=0.3$, $\alpha_6=0.23$, $\omega_6=0.07$, $\alpha_7=0.07$, $\omega_7=0.001$, $\alpha_8=0.08$, $\omega_8=0.02$, $\alpha_9=0.15$, $\omega_9=0.07$, $\alpha_{10}=0.02$, and $\omega_{10}=0.05$. The variations of the original and estimated parameters along with the synchronization error are shown in Fig. 1(c) for coupling $k=25$, which satisfies condition (6). In this way one can estimate multiple parameters using a single scalar chaotic time series just satisfying the sufficient condition (6) for synchronization. We have numerically estimated 50 parameters at high coupling strength.

In the above discussion, we have shown the method of parameter estimation using a synchronized delayed system. Now in this part we propose an algorithm by which piecewise and binary messages and analog and chaotic signals can be transmitted. The algorithm can be written as follows:

$$\dot{x} = -ax + \sum_{i=1}^{N} m_i f(x_{\tau_i}), \tag{16}$$

$$\dot{y} = -ay + \sum_{i=1}^{N} n_i f(y_{\tau_i}) + k(x-y), \tag{17}$$

$$\dot{n}_i = \frac{1}{\gamma}(x-y)\sum_{i=1}^{N} f(x_{\tau_i}), \tag{18}$$

where γ is the adaptive parameter.

First we choose m_1 as a digital message in the form

$$m_1 = \begin{cases} 4.0, & t < 500, \\ 5.0, & 500 \leq t < 1000, \\ 4.0, & 1000 \leq t < 1500, \\ 5.0, & 1500 \leq t < 2000. \end{cases} \tag{19}$$

The value of the modulation parameter m_1 is switched between 4.0 and 5.0 in the transmitter according to the nature of the digital message. Due to this switching in the value of the modulation parameter, the response system is driven by x corresponding to $m_1=4.0$, if the transmitted message bit is 0, or by x corresponding to $m_1=5.0$, if the transmitted message bit is 1. The original and estimated signals and synchronization error are shown in Fig. 2(a) by solid, dotted, and dashed lines, respectively, for coupling $k=3$ and $\gamma=0.01$.

Next we choose m_1 as a binary message in the form

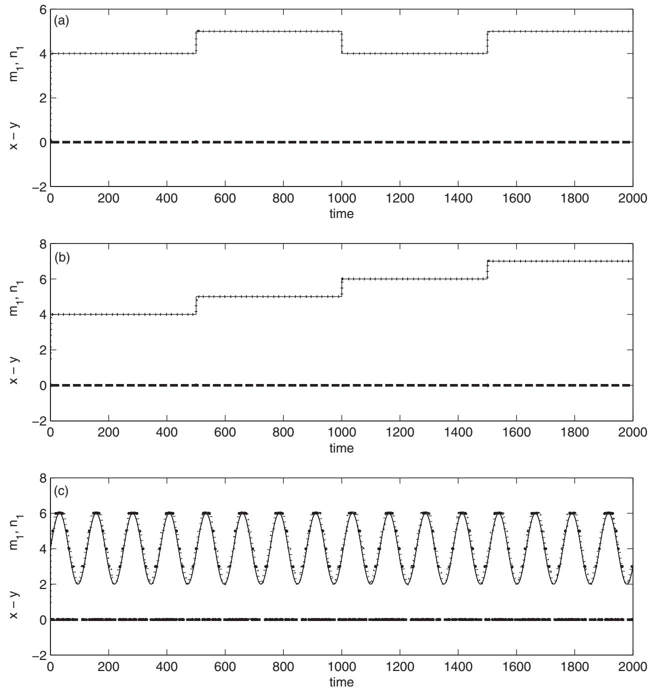

FIG. 2. Variation of synchronization error (dashed lines), original signal (solid lines), and estimated signal (dotted lines) in the form of a (a) digital message in the form (19), (b) binary message (20), and (c) sinusoidal signal $m_1(t)=4+\alpha\sin(\omega t)$.

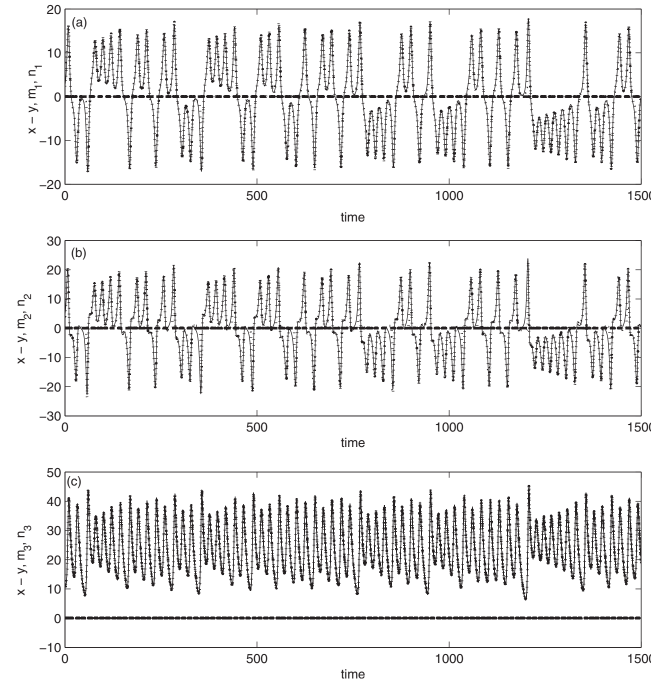

FIG. 3. Variation of synchronization error (dashed lines), original chaotic signal (solid lines), and estimated chaotic signal (dotted lines) of Lorenz system with (a) $m_1(t)=u(t)$, (b) $m_2(t)=v(t)$, and (c) $m_3(t)=w(t)$.

$$m_1 = \begin{cases} 4.0, & t < 500, \\ 5.0, & 500 \leqslant t < 1000, \\ 6.0, & 1000 \leqslant t < 1500, \\ 7.0, & 1500 \leqslant t < 2000. \end{cases} \tag{20}$$

The original signal, estimated signal, and synchronization error are shown in Fig. 2(b) for coupling $k=5$ and $\gamma=0.01$.

We take m_1 as a sinusoidal signal in the form

$$m_1(t) = 4 + \alpha\sin(\omega t), \tag{21}$$

where α and ω are the amplitude and frequency of the signal, respectively. The original and estimated signal and synchronization error are shown in Fig. 2(c) for coupling $k=5$ and $\gamma=0.005$, $\alpha=2$, $\omega=0.05$.

Finally, we show that it is also possible to estimate the chaotic signals coming from a different trajectory. We choose $m_1(t)$, $m_2(t)$, and $m_3(t)$ as the chaotic time series of a Lorenz system $[\dot{u}=\sigma(v-u), \dot{v}=ru-v-uw, \dot{w}=uv-bw, \sigma=10, r=28, b=8/3]$, i.e., $m_1(t)=u(t)$, $m_2(t)=v(t)$, and $m_3(t)=w(t)$. The original chaotic signals, estimated chaotic signals, and synchronization error are shown in Figs. 3(a)–3(c) for coupling $k=5$ and $\gamma=0.001$. From the figure we can see that the signals are well recovered even through they are complex and chaotic in nature. In this case we have taken three different parameters m_1, m_2, m_3 to estimate the chaotic signals.

IV. CONCLUSIONS

The advantages of the method are the following. (a) It is able to estimate all parameters of a chaotic multiple-time-delay system. (b) More than one piece of information can be transmitted through a chaotic time series. (c) The implementation cost can be reduced. (d) The method can be easily implemented using a simple analog circuit in practical communication applications. Each of these advantages is distinct from the existing methods [2–6,10–13], where more than one time series is needed to estimate multiple parameters of chaotic systems. In addition, the approach presented here is different from those techniques based on the adaptive control approach [1], the active-passive-decomposition method [2], autosynchronization [3], the error minimization strategy [2], statistical methods [4], iterative methods [5], and the least-squares approach [6].

Past results indicate that the method proposed here could be more easily applicable in experiments. The proposed method is very applicable in experiments where only a single output is available. The resulting systems can be easily and inexpensively implemented using a simple analog circuit, making this approach feasible for practical communication applications. This is a unified treatment by which one can transmit more than one signal (digital, sinusoidal, and chaotic) at a time by using only one scalar chaotic time series. Therefore, this adaptive scheme for synchronization-based multiple-parameter estimation provides a unified treatment for a large class of general multiple-time-delayed systems with modulated delay time.

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